Applied Mathematics (JFL201 (4:0:0), January – April 2024)

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Matrix Analysis: Linear vector space and Matrices; Inner product and Gram-Schmidt orthonormalization; Similarity transformation; Canonical forms; Hermitian, Unitary and Normal matrices; Eigenvalues and Eigenvectors; LU, Cholesky and Singular-Value decompositions; Positive definite matrices and Optimization.

Complex Variables and Integral Transform Techniques: Complex number and their properties: limits, continuity, and complex differentiation. Analytic functions: Cauchy-Riemann equations and potential flows.
Multivalued functions: Branch points and Branch cuts; Riemann sheet.
Power Series: Taylor and Laurent series expansions.
Complex Integration: Cauchy's theorem; Cauchy's integral formula and its derivatives; Cauchy residue theorem and contour integration.
Infinite series sum, Analytic Continuation and Riemann $\zeta(p)$-function
Fourier and Laplace transforms, and their inverses; Convolution theorem and the Parseval formula.

Ordinary Differential Equations: Homogeneous and inhomogeneous linear differential equations: Green's Function, etc.
Canonical nonlinear differential equations: Bernoulli, Riccati equations, etc.
Regular and Irregular Singular Points: Fuchsian DEs and Series Solution.
Möbius transformation: From Riemann DE to Gauss/Hypergeometric DE.
Confluence of RSPs: Confluent Hypergeometric, Bessel, Jacobi, Hermite and Laguerre equations; Orthogonality and related recurrence relations.
Adjoint operators and Strum-Liouville problem.
Eigenfunction Expansion and Generalized Fourier series.
From Helmholtz PDE to Spherical Bessel and Associated Legendre Equations, and the Spherical Harmonics.

Partial Differential Equations: Classification of PDEs: Heat equation; Wave equation; Laplace equation.
Separation of variables and related eigenfunction expansion methods.
Infinite Domain and Continuous Spectra.
Green's function technique for inhomogeneous equations, with applications in Mechanics
First-order quasi-linear PDEs and the method of characteristics.
Canonical Nonlinear PDEs: Burgers$\leftrightarrow$Heat, KdV and Ginzburg-Landau equations and their applications.
Integrable PDEs and the Fokas Method (Unified Transform)

Suggested Reading

1. Ablowitz, M.J. and Fokas, A.S. (1998) *Complex Variables: Introduction and Applications*. Cambridge University Press.
2. Birkhoff, G. and Rota, G.-C. (1989) *Ordinary Differential Equations*. John Wiley & Sons.
3. Gustafson, K.E. (1980) *An Introduction to PDEs and Hilbert Space Methods*. Dover Publications.
4. Horn, R.A. and Johnson, C.R. (1985) *Matrix Analysis*. Cambridge University Press.