Applied Mathematics (JF201, 3 Credits; Aug-Dec 2017)

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Vectors and Tensors: Review of vector and tensor analysis. Coordinate transformation and Jacobian. Local theory of curves: Frenet-Serret equations. Gauss divergence theorem and Stokes' theorem. Irrotational and Solenoidal vector fields; Helmholz' representation. Non-Cartesian tensors: Metric tensor; Parallel transport and Christoffel Symbols; Covariant derivative.

Matrix Analysis: Linear vector space and Matrices; Inner product and Gram-Schmidt orthonormalization; Similarity transformation; Canonical forms; Hermitian, Unitary and Normal matrices; Eigenvalues and Eigenvectors; Positive definite matrices and Optimization; LU, Cholesky and Singular-Value decompostions. Functions of matrices.

Complex Variables: Complex number and their properties: limits, continuity, and complex differentiation. Analytic functions: Cauchy-Riemann equations and potential flows. Multivalued functions: Branch points and Branch cuts; Riemann sheet. Power Series: Taylor and Laurent series expansions. Complex Integration: Cauchy's theorem; Cauchy's integral formula and its derivatives; Cauchy residue theorem and contour integration.

Integral Transform Techniques: Fourier and Laplace transforms, and their inverses; Convolution theorem and the Parseval formula; Applications to differential equations.

Ordinary Differential Equations: Methods to solve homogeneous and inhomogeneous linear differential equations: Green's Function, etc. Canonical nonlinear differential equations: Bernoulli, Riccati equations, etc. Regular and Irregular Singular Points: Fuchsian DEs and Series Solution. Möbius transformation: From Riemann P-equation to Hypergeoemtric DE. Confluence of RSPs: Confluent Hypergeometric, Bessel, Legendre, Hermite and Laguerre equations; Orthogonality and related recurrence relations. From Helmholtz PDE to Spherical Bessel and Associated Legendre Equations, and the Spherical Harmonics. Adjoint operators and Strum-Liouville problem. Eigenfucntion Expansion and Generalized Fourier series.

Partial Differential Equations: Classification of PDEs: Heat equation; Wave equation; Laplace equation. Separation of variables and related eigenfunction expansion methods. Infinite Domain and Continuous Spectra. Green's function technique for inhomogeneous equations.

Nonlinear Dynamics: Nonlinear ODEs: Phase plane analysis, critical points, stability criteria and bifurcation. Logistic Map: Bifurcation and chaos. Solutions of canonical Nonlinear PDEs for dissipative and dispersive waves: Burger and KdV equations.

1. Ablowitz, M.J. and Fokas, A.S. (1998) *Complex Variables: Introduction and Applications*, Cambridge University Press.

2. Arfken, G.B. and Weber, H.J. (2001) *Mathematical Methods for Physicists*, Academic Press.

3. Aris, R. (1962) *Vectors, Tensors and the Basic Equations of Fluid Mechanics*, Dover Publications.

4. Bender, C.M. and Orszag, S.A. (1978) *Advanced Mathematical Methods for Scientists and Engineers*, McGraw-Hill.

5. Birkhoff, G. and Rota, G.-C. (1989) *Ordinary Differential Equations*, John Wiley & Sons.

6. Drazin, P.G. (1992) *Nonlinear Systems*, Cambridge University Press.

7. Gustafson, K.E. (1980) *An Introduction to PDEs and Hilbert Space Methods*, Dover Publications.

8. Horn, R.A. and Johnson, C.R. (1985) *Matrix Analysis*, Cambridge University Press.