Course on Solid Mechanics

The course offers an introduction to solid mechanics. It begins with an overview of continuum mechanics progressing to the formulation of the general boundary value problems for linear elasticity. We explore the conditions for well-posedness and study the treatment of singularities and modelling of defects in a linear elastic medium. We transition to finite strain elasticity studying the variational framework for nonlinear elasticity, covering notions of material symmetry, role of convexity, interfaces. The course concludes with an introduction to mechanics of slender structures including Cosserat rod theory and membrane mechanics.

The prerequisites for this course are exposure to basic linear algebra and multivariable calculus.

Syllabus

- Mathematical Preliminaries:
  - Tensor analysis: differential operators
  - Integral laws
  - Poincare's lemma
- Kinematics:
  - Deformation: configurations, deformation gradient, strain and rotation
  - Linearised strain: compatibility conditions
  - Geometric rigidity
- Balance Laws
  - Linear and angular momentum balance
  - Stress tensor, local forms of balance laws
  - Different stress representations
- Linear Elasticity
  - Constitutive relation, material constants, material stability
  - Boundary value problem, uniqueness of solution
  - Semi-inverse method
  - Singularities: Kelvin's problem, Green's functions
  - Topological defects: dislocations, disclinations
- Non-linear Elasticity
  - Elastic energy, variational formulation
  - Material symmetry, representation for isotropic response
  - Local constraints on deformations

  - Some boundary value problems
  - Stability, notions of convexity
  - Compatibility and equilibrium conditions for discontinuous deformation gradients
- Slender elastic bodies
  - Cosserat rod theory
  - Membrane theory

References

- P. Chadwick, *Continuum Mechanics: Concise Theory and Problems*, Courier Corporation 1999
- M.E. Gurtin, *An Introduction to Continuum Mechanics*, Academic Press, 1981
- M.E. Gurtin, *The Linear Theory of Elasticity*, in Mechanics of Solids-Volume 2, edited by C. Truesdell
- D. J. Steigmann, *Finite Elasticity Theory*, Oxford University Press, 2017
- R.W. Ogden, *Non-Linear Elastic Deformations*, Courier Corporation, 1997