Differential Geometry with Applications to Mechanics

Fundamental concepts from differential geometry play a central role in our understanding of the mechanics of both solids and fluids. This course will offer an introduction to some of these ideas and their application to mechanics.

The prerequisites of the course are a basic understanding of linear algebra (vectors, matrices etc.) and basic calculus.

Syllabus

- Manifolds
  - Differentiable manifolds
  - Tangent and Cotangent Spaces
  - Tensors
  - Differential Forms and Exterior Derivative
  - Integration on Manifolds, Stokes' Theorem
- Three Dimensional Manifolds
  - Curvilinear Coordinates
  - Metric Tensor and Covariant Derivative
  - Riemann Curvature Tensor
  - Three Dimensional Euclidean Space ($\mathbb{E}^3$)
  - Three Dimensional Elasticity in Curvilinear Coordinates
- Curves in $\mathbb{E}^3$
  - Regular Curves
  - Length, Curvature and Torsion
  - Orthonormal Frames on Curves
  - Global (Topological) Properties of Curves: Knots
  - Cosserat Theory of Elastic Rods
- Surfaces in $\mathbb{E}^3$
  - Smooth Surfaces
  - Fundamental Forms
  - Compatibility of Forms (Gauss Theorem and Codazzi-Mainardi Equations)
  - Global (Topological) Properties of Surfaces: Gauss-Bonnet Theorem, Euler Characteristic
- Theory of Elastic Shells
  - Kinematics, Strain Measures
  - Equilibrium Equations

- Variational Formulation
- von-Kármán Plate Theory
- Geometric Modelling of Defects

- Fluid Mechanics
  - Flows on Curved Geometries
  - Vortex Curves
  - Defects in Liquid Crystals

References

- M.P. Do Carmo, *Differential Geometry of Curves and Surfaces.*
- M.P. Do Carmo, *Differential Forms and Applications.*
- P.G. Ciarlet, *An introduction to Differential Geometry with Applications to Elasticity.*
- M. Epstein, *The Geometrical Language of Continuum Mechanics.*
- B. Audoly and Y.Pomeau, *Elasticity and Geometry.*
- V.I. Arnold and B.A. Khesin *Topological Methods in Hydrodynamics*